

which it is approximately plastic. The reverse process has been observed in the shock compression of MgO [Ahrens, 1966] and other materials.

The interpretation of the present observations is aided by comparisons between the measured velocities and various extrapolations of the longitudinal elastic velocity  $V_L$  and the bulk sound speed  $C$  (which are discussed in detail below). The measured and extrapolated velocities are plotted against pressure in Figure 4. The shots with the greatest mirror-sample separation  $z$ , which were the most sensitive to contrasts in free surface velocities, yielded the highest measured rarefaction velocities. These velocities are close to the extrapolations of  $V_L$ . The shots with smaller  $z$  yielded considerably lower measured rarefaction velocities closer to  $C$  for the smallest value of  $z$  (A258). This finding suggests that the arrangement in the latter shots was not sensitive enough to detect the initial elastic decompression observed in the previous shots, so that the observed edge effect was produced mainly by the hydrodynamic (or plastic) decompression. (The half error bars shown in Figure 4 were obtained by picking points on the records where edge effects definitely existed. The best estimates of the extent of the edge effect and the lower bounds are indicated by the inner and outer parts of tick marks in Figure 3, respectively. Obviously, upper bounds cannot be estimated.)

The major uncertainty in the above interpretation of the data is whether the profile of the free surface when it strikes the witness mirror accurately reflects the conditions at the free surface when the shock wave reached it or whether the free surface undergoes some distortion during transit to the witness mirror. Suspicions of the latter effect were in fact the original reason for varying the mirror-sample spacing. Finite difference computations are being carried out in an attempt to verify the source of the observed effects.

The extrapolations of  $V_L$  and  $C$  in Figure 4 are based on the elastic moduli and their pressure and temperature derivatives determined ultrasonically by Spetzler [1970]. They are third- and fourth-order finite strain extrapolations in terms of the Eulerian and Lagrangian strain parameters  $\epsilon$  and  $\eta$  and include the thermal effects in the 'quasi-harmonic' approxi-

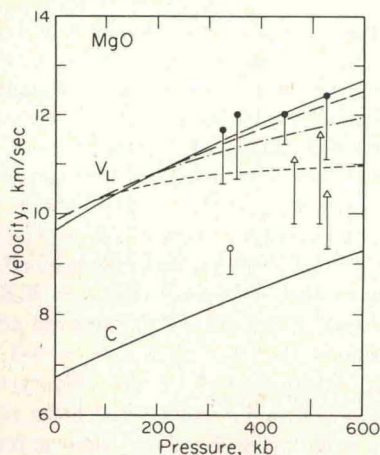


Fig. 4. Measured rarefaction velocities and extrapolated longitudinal ( $V_L$ ) and bulk ( $C$ ) sound velocities in the shocked state. Mirror-sample separations were 0.76 mm (solid circles), 0.13 mm (open circle), and 0.25 mm (triangles). Extrapolations are for polycrystal,  $\xi_{ij} = -1$  (solid); single-crystal [110] direction,  $\xi_{ij} = -1$  (long-dashed);  $\xi_{120} = -10$  (dash-dot); and third-order Lagrangian (short-dashed).

mation [Davies, 1972; unpublished manuscript, 1973]. In terms of  $\epsilon$ , the fourth-order finite strain formula for the effective elastic moduli  $c_{ij}$  (Voigt notation) from which the velocities are calculated has the form [Davies, 1972; unpublished manuscript, 1973]

$$c_{ij} = \rho_0(1 - 2\epsilon)^{7/2} \cdot (r_{ij}^0 + r_{ij}^1\epsilon + \frac{1}{2}r_{ij}^2\epsilon^2) - P\Delta_{ij} \quad (3)$$

where the  $r_{ij}^n$  are parameters,  $P$  is the pressure, and, in the case of cubic symmetry,

$$\Delta_{11} = -3 \quad \Delta_{12} = -1 \quad \Delta_{44} = -1 \quad (4)$$

The corresponding expression for the pressure is [Davies, 1972, 1973]

$$P = -\frac{1}{3}\rho_0(1 - 2\epsilon)^{5/2}(c_0 + c_1\epsilon + c_2\epsilon^2 + c_3\epsilon^3) \quad (5)$$

The strain parameter  $\epsilon$  is related to density by

$$\epsilon = \frac{1}{2}[1 - (\rho/\rho_0)^{2/3}] \quad (6)$$

The parameters  $c_n$  and  $r_{ij}^n$  in (3) and (5) are related to the  $c_{ij}$  and their pressure derivatives evaluated at  $\rho_0$ . These relations are given in the appendix. Analogous equations in terms of the Lagrangian strain  $\eta$  can also be derived [Davies,

1972, 1973, unpublished manuscript, 1973; Thomsen, 1970, 1972].

The full fourth-order versions of (3) and (5) require the second pressure derivatives of the effective elastic moduli  $c_{ij}''$  (see equation A7). These have not been measured, but the present results can be used to place constraints on them. The second pressure derivative of the bulk modulus  $K''$  of MgO has been determined from shock wave data to be such that  $\zeta = K_0 K_0'' = -1$  [Davies, 1973]. As a first approximation it was assumed that  $\zeta_{ij} = K_0 c_{ij}'' = -1$ . The resulting extrapolations of the polycrystalline average velocities  $V_L$  and  $C$  are shown in Figure 4 as solid curves. Note that these extrapolations include corrections for the increasing temperature along the Hugoniot (Table 1). This extrapolation of  $V_L$  gives good agreement with the measurements obtained by using the largest mirror-sample separation (solid circles). The appropriate longitudinal velocity in the single-crystal samples is that for the [110] crystallographic direction, since  $\tan \alpha \simeq 1$  (Table 1) and the crystal faces were (100) planes. This velocity  $V_{110}$  is shown extrapolated in Figure 4 by the long-dashed curve. It is quite close to the polycrystalline velocity, and the distinction is hardly warranted by the accuracy of the data.

The bounds on the  $c_{ij}''$  can be estimated from the extrapolation of  $V_{110}$  by using  $\zeta_{110} = \frac{1}{2}(\zeta_{11} + \zeta_{12} + 2\zeta_{44}) = -10$ , which is shown as the dash-dot curve. We may thus deduce that  $\zeta_v = -1 \pm 15$  where  $\zeta_v$  is the appropriate combination of  $\zeta_{ij}$  (i.e., corresponding to the polycrystalline or the [110] single-crystal velocity).

In view of the current discussion of the relative empirical merits of the  $\eta$  and  $\epsilon$  strain measures [e.g., Ahrens and Thomsen, 1972], third-order  $\eta$  and  $\epsilon$  extrapolations of  $V_L$  were compared with the present data. The third-order  $\epsilon$  extrapolation was found to be quite close to the fourth-order  $\epsilon$  extrapolation with  $\zeta_{ij} = -1$  and is not shown in Figure 4. The third-order  $\eta$  extrapolation is shown by the short-dashed curve. It is seen to give a poorer fit to the solid circles, which are here interpreted as the most accurate measurements of  $V_L$ .

In conclusion, our data suggest that the two-stage elastoplastic model for solids [e.g., Kusubov and van Thiel, 1969] applies to MgO under decompression. This implies also that

MgO remains in the solid state to at least 500 kb and 600°K under shock compression. Our results yield the approximate bounds  $\zeta_v = -1 \pm 15$  as determined from Eulerian finite strain equations. Further refinement of this technique should allow more accurate determinations of the high-pressure, high-temperature elastic properties of materials of geophysical interest.

#### APPENDIX

The parameters  $c_n$  in (5) are related to the  $r_{ij}^n$  in (3):

$$c_1 = 3(r_{11}^0 + 2r_{12}^0) \quad (\text{A1})$$

$$c_2 = \frac{3}{2}(r_{11}^1 + 2r_{12}^1) \quad (\text{A2})$$

$$c_3 = \frac{1}{2}(r_{11}^2 + 2r_{12}^2) \quad (\text{A3})$$

By evaluating (5) at  $\rho_0$ , we see that

$$c_0 = -3P_0/\rho_0 \quad (\text{A4})$$

where  $P_0$  is the pressure at  $\rho_0$ , and  $P_0$  depends on the temperature  $T$ .

By differentiating (3), the parameters  $r_{ij}^n$  can be related to  $P_0$  and the  $c_{ij}$  and their pressure derivatives  $c_{ij}'$ ,  $c_{ij}''$ , etc., evaluated at  $\rho_0$ :

$$r_{ij}^0 = (c_{ij} + P_0 \Delta_{ij})/\rho_0 \quad (\text{A5})$$

$$r_{ij}^1 = -3K_0(c_{ij}' + \Delta_{ij})/\rho_0 + 7r_{ij}^0 \quad (\text{A6})$$

$$r_{ij}^2 = 9K_0^2 c_{ij}''/\rho_0 - 3K_0'(r_{ij}^1 - 7r_{ij}^0) + 16r_{ij}^1 - 49r_{ij}^0 \quad (\text{A7})$$

where  $K = \frac{1}{3}(c_{11} + 2c_{12})$  is the bulk modulus.

Thermal effects in the finite strain equations 3 and 5 can be described through the parameters  $r_{ij}^n$  and  $c_n$ . Expressions for the temperature dependence of these parameters have been derived [Davies, 1972, 1973, unpublished manuscript, 1973] from the quasi-harmonic approximation of lattice dynamics [Leibfried and Ludwig, 1961]. These expressions require for their evaluation the thermal expansion coefficient, the temperature derivatives of the elastic moduli, and the specific heat of the material.

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